

# Dependence of Optical Transition Energies on Structure for Single-Walled Carbon Nanotubes in Aqueous Suspension: An Empirical Kataura Plot

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## ABSTRACT

Spectrofluorimetric data for identified single-walled carbon nanotubes in aqueous SDS suspension have been accurately fit to empirical expressions. These are used to obtain the first model-independent prediction of first and second van Hove optical transitions as a function of structure for a wide range of semiconducting nanotubes. To allow for convenient use in support of spectral studies, the results are presented in equation, graphical, and tabular forms. These empirical findings differ significantly from Kataura plots computed using a simple tight-binding model. It is suggested that the empirically based results should be used in preference to conventional model-based predictions in spectroscopic nanotube research.

**Introduction.** One of the most intriguing and potentially useful features of single-walled carbon nanotubes (SWNT) is the sensitivity of their electronic and optical properties to physical structure.<sup>1</sup> A nanotube's diameter ( $d_i$ ) and chiral angle ( $\alpha$ ) are uniquely related to a pair of integers ( $n, m$ ) that describe its construction as a rolled-up graphene sheet. Except for very small diameter nanotubes, it is well known that structures for which  $n - m$  is evenly divisible by 3 display metallic or semimetallic behavior, whereas other tubes only slightly different in structure have significant band gaps and show semiconducting behavior.<sup>2</sup> The quasi-one-dimensionality of SWNT gives sharp van Hove peaks in the density of electronic states. Optical properties of SWNT are dominated by transitions between corresponding van Hove peaks on opposite sides of the Fermi level. Knowing the energies of these  $E_{ii}$  van Hove transitions for specific ( $n, m$ ) structures is important not only for absorption spectroscopy but also for the widely used method of resonance Raman spectroscopy in which matches between the incident or scattered light frequencies and van Hove transitions expose vibrational frequencies of selected nanotubes.<sup>3–5</sup> Nanotube investigators using optical methods are commonly guided by Kataura plots of  $E_{ii}$  transition energies vs tube diameter.<sup>6</sup> In the absence of detailed experimental data, these have been generated from parametrized model calculations of SWNT electronic structure.

Recent breakthroughs in SWNT spectroscopy now allow the construction of experimentally based Kataura-type plots. In semiconducting SWNT, the lowest-energy van Hove transition (at  $E_{11}$ ) corresponds to absorption or luminescence directly across the band gap. Since the discovery of band-gap photoluminescence from isolated SWNT in bulk aqueous suspensions,<sup>7</sup> it has become possible to use spectrofluorimetry to explore the  $E_{11}$  and  $E_{22}$  transitions of a sizable number of semiconducting SWNT species. As was reported earlier from this laboratory, distinct first and second van Hove transition wavelengths have been observed for 33 semiconducting SWNT in a surfactant-suspended bulk sample, and each of these species has been assigned specific ( $n, m$ ) structural indices.<sup>8</sup> Spectral transitions of six additional SWNT species were reported by Lebedkin et al. in a study of slightly larger diameter nanotubes in a different surfactant.<sup>9</sup> Because the identified nanotubes span a rather wide range of structures (ranging from 0.62 to 1.41 nm in diameter and from 3 to 28° in chiral angle), their measured transition frequencies can be used to anchor a robust empirical fitting function that allows extrapolation beyond the set of measured data. This provides the basis for a model-independent, empirically based Kataura plot that gives reliable predictions of optical transition frequencies versus ( $n, m$ ) for semiconducting SWNT in aqueous surfactant suspensions. To enable easy use of the new predictions, we present them below as empirical functions of diameter and chiral angle, as graphs, and as tabulated values for more than 100 ( $n, m$ ) structures.

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**Table 1.** Structures and First and Second van Hove Optical Transitions<sup>a</sup> for Semiconducting SWNT Structures with Diameters between 0.48 and 2.0 nm

| (n, m)   | $d_t$<br>(nm) | $\alpha$<br>(deg) | mod <sup>b</sup> | $\lambda_{11}$<br>(nm) | $\bar{\nu}_{11}$<br>(cm <sup>-1</sup> ) | $E_{11}$<br>(eV) <sup>c</sup> | $\lambda_{22}$<br>(nm) | $\bar{\nu}_{22}$<br>(cm <sup>-1</sup> ) | $E_{22}$<br>(eV) | (n, m)   | $d_t$<br>(nm) | $\alpha$<br>(deg) | mod <sup>b</sup> | $\lambda_{11}$<br>(nm) | $\bar{\nu}_{11}$<br>(cm <sup>-1</sup> ) | $E_{11}$<br>(eV) <sup>c</sup> | $\lambda_{22}$<br>(nm) | $\bar{\nu}_{22}$<br>(cm <sup>-1</sup> ) | $E_{22}$<br>(eV) |
|----------|---------------|-------------------|------------------|------------------------|---|-------------------------------|------------------------|---|------------------|----------|---------------|-------------------|------------------|------------------------|---|-------------------------------|------------------------|---|------------------|
| (4, 3)   | 0.483         | 25.28             | 1                | 700                    | 14 283                                  | 1.771                         | 398                    | 25 147                                  | 3.118            | (14, 12) | 1.789         | 27.46             | 2                | 2059                   | 4856                                    | 0.602                         | 1181                   | 8470                                    | 1.050            |
| (5, 3)   | 0.556         | 21.79             | 2                | 720                    | 13 884                                  | 1.721                         | 522                    | 19 147                                  | 2.374            | (14, 13) | 1.857         | 28.78             | 1                | 2141                   | 4672                                    | 0.579                         | 1208                   | 8278                                    | 1.026            |
| (5, 4)   | 0.620         | 26.33             | 1                | 835                    | 11 974                                  | 1.485                         | 483                    | 20 697                                  | 2.566            | (15, 1)  | 1.232         | 3.20              | 2                | 1426                   | 7011                                    | 0.869                         | 920                    | 10 864                                  | 1.347            |
| (6, 1)   | 0.521         | 7.59              | 2                | 653                    | 15 323                                  | 1.900                         | 632                    | 15 828                                  | 1.962            | (15, 2)  | 1.278         | 6.18              | 1                | 1622                   | 6165                                    | 0.764                         | 822                    | 12 163                                  | 1.508            |
| (6, 2)   | 0.572         | 13.90             | 1                | 894                    | 11 183                                  | 1.387                         | 418                    | 23 900                                  | 2.963            | (15, 4)  | 1.377         | 11.52             | 2                | 1589                   | 6294                                    | 0.780                         | 986                    | 10 140                                  | 1.257            |
| (6, 4)   | 0.692         | 23.41             | 2                | 873                    | 11 452                                  | 1.420                         | 578                    | 17 312                                  | 2.146            | (15, 5)  | 1.431         | 13.90             | 1                | 1752                   | 5708                                    | 0.708                         | 921                    | 10 858                                  | 1.346            |
| (6, 5)   | 0.757         | 27.00             | 1                | 976                    | 10 244                                  | 1.270                         | 566                    | 17 667                                  | 2.190            | (15, 7)  | 1.546         | 18.14             | 2                | 1779                   | 5621                                    | 0.697                         | 1064                   | 9396                                    | 1.165            |
| (7, 0)   | 0.556         | 0.00              | 1                | 962                    | 10 397                                  | 1.289                         | 395                    | 25 318                                  | 3.139            | (15, 8)  | 1.606         | 20.03             | 1                | 1907                   | 5245                                    | 0.650                         | 1035                   | 9663                                    | 1.198            |
| (7, 2)   | 0.650         | 12.22             | 2                | 802                    | 12 468                                  | 1.546                         | 626                    | 15 977                                  | 1.981            | (15, 10) | 1.730         | 23.41             | 2                | 1987                   | 5032                                    | 0.624                         | 1156                   | 8650                                    | 1.072            |
| (7, 3)   | 0.706         | 17.00             | 1                | 992                    | 10 083                                  | 1.250                         | 505                    | 19 820                                  | 2.457            | (15, 11) | 1.795         | 24.92             | 1                | 2086                   | 4793                                    | 0.594                         | 1158                   | 8635                                    | 1.071            |
| (7, 5)   | 0.829         | 24.50             | 2                | 1024                   | 9768                                    | 1.211                         | 645                    | 15 496                                  | 1.921            | (15, 13) | 1.927         | 27.64             | 2                | 2207                   | 4532                                    | 0.562                         | 1259                   | 7942                                    | 0.985            |
| (7, 6)   | 0.895         | 27.46             | 1                | 1120                   | 8930                                    | 1.107                         | 648                    | 15 441                                  | 1.914            | (15, 14) | 1.994         | 28.86             | 1                | 2287                   | 4373                                    | 0.542                         | 1288                   | 7767                                    | 0.963            |
| (8, 0)   | 0.635         | 0.00              | 2                | 776                    | 12 886                                  | 1.598                         | 660                    | 15 146                                  | 1.878            | (16, 0)  | 1.270         | 0.00              | 1                | 1623                   | 6163                                    | 0.764                         | 815                    | 12 264                                  | 1.521            |
| (8, 1)   | 0.678         | 5.82              | 1                | 1041                   | 9603                                    | 1.191                         | 471                    | 21 226                                  | 2.632            | (16, 2)  | 1.357         | 5.82              | 2                | 1561                   | 6405                                    | 0.794                         | 984                    | 10 162                                  | 1.260            |
| (8, 3)   | 0.782         | 15.30             | 2                | 952                    | 10 508                                  | 1.303                         | 665                    | 15 029                                  | 1.863            | (16, 3)  | 1.405         | 8.44              | 1                | 1746                   | 5728                                    | 0.710                         | 898                    | 11 139                                  | 1.381            |
| (8, 4)   | 0.840         | 19.11             | 1                | 1111                   | 8997                                    | 1.116                         | 589                    | 16 981                                  | 2.105            | (16, 5)  | 1.508         | 13.17             | 2                | 1732                   | 5775                                    | 0.716                         | 1055                   | 9480                                    | 1.175            |
| (8, 6)   | 0.966         | 25.28             | 2                | 1173                   | 8525                                    | 1.057                         | 718                    | 13 928                                  | 1.727            | (16, 6)  | 1.564         | 15.30             | 1                | 1884                   | 5307                                    | 0.658                         | 1000                   | 9999                                    | 1.240            |
| (8, 7)   | 1.032         | 27.80             | 1                | 1265                   | 7908                                    | 0.981                         | 728                    | 13 727                                  | 1.702            | (16, 8)  | 1.680         | 19.11             | 2                | 1925                   | 5194                                    | 0.644                         | 1138                   | 8788                                    | 1.090            |
| (9, 1)   | 0.757         | 5.21              | 2                | 912                    | 10 964                                  | 1.359                         | 691                    | 14 466                                  | 1.794            | (16, 9)  | 1.741         | 20.82             | 1                | 2046                   | 4887                                    | 0.606                         | 1115                   | 8968                                    | 1.112            |
| (9, 2)   | 0.806         | 9.83              | 1                | 1138                   | 8790                                    | 1.090                         | 551                    | 18 155                                  | 2.251            | (16, 11) | 1.867         | 23.90             | 2                | 2134                   | 4685                                    | 0.581                         | 1233                   | 8111                                    | 1.006            |
| (9, 4)   | 0.916         | 17.48             | 2                | 1101                   | 9086                                    | 1.126                         | 722                    | 13 843                                  | 1.716            | (16, 12) | 1.932         | 25.28             | 1                | 2231                   | 4483                                    | 0.556                         | 1238                   | 8077                                    | 1.001            |
| (9, 5)   | 0.976         | 20.63             | 1                | 1241                   | 8055                                    | 0.999                         | 672                    | 14 883                                  | 1.845            | (17, 0)  | 1.350         | 0.00              | 2                | 1552                   | 6443                                    | 0.799                         | 984                    | 10 167                                  | 1.261            |
| (9, 7)   | 1.103         | 25.87             | 2                | 1322                   | 7567                                    | 0.938                         | 793                    | 12 610                                  | 1.563            | (17, 1)  | 1.391         | 2.83              | 1                | 1744                   | 5733                                    | 0.711                         | 886                    | 11 289                                  | 1.400            |
| (9, 8)   | 1.170         | 28.05             | 1                | 1410                   | 7093                                    | 0.879                         | 809                    | 12 362                                  | 1.533            | (17, 3)  | 1.483         | 7.99              | 2                | 1699                   | 5886                                    | 0.730                         | 1050                   | 9525                                    | 1.181            |
| (10, 0)  | 0.794         | 0.00              | 1                | 1156                   | 8652                                    | 1.073                         | 537                    | 18 606                                  | 2.307            | (17, 4)  | 1.533         | 10.33             | 1                | 1873                   | 5340                                    | 0.662                         | 974                    | 10 263                                  | 1.272            |
| (10, 2)  | 0.884         | 8.95              | 2                | 1053                   | 9493                                    | 1.177                         | 737                    | 13 574                                  | 1.683            | (17, 6)  | 1.641         | 14.56             | 2                | 1875                   | 5332                                    | 0.661                         | 1125                   | 8886                                    | 1.102            |
| (10, 3)  | 0.936         | 12.73             | 1                | 1249                   | 8006                                    | 0.993                         | 632                    | 15 834                                  | 1.963            | (17, 7)  | 1.697         | 16.47             | 1                | 2019                   | 4952                                    | 0.614                         | 1079                   | 9264                                    | 1.149            |
| (10, 5)  | 1.050         | 19.11             | 2                | 1249                   | 8006                                    | 0.993                         | 788                    | 12 695                                  | 1.574            | (17, 9)  | 1.816         | 19.93             | 2                | 2072                   | 4827                                    | 0.599                         | 1212                   | 8248                                    | 1.023            |
| (10, 6)  | 1.111         | 21.79             | 1                | 1377                   | 7262                                    | 0.900                         | 754                    | 13 262                                  | 1.644            | (17, 10) | 1.877         | 21.49             | 1                | 2188                   | 4571                                    | 0.567                         | 1195                   | 8367                                    | 1.037            |
| (10, 8)  | 1.240         | 26.33             | 2                | 1470                   | 6805                                    | 0.844                         | 869                    | 11 502                                  | 1.426            | (18, 1)  | 1.470         | 2.68              | 2                | 1682                   | 5944                                    | 0.737                         | 1048                   | 9543                                    | 1.183            |
| (10, 9)  | 1.307         | 28.26             | 1                | 1556                   | 6428                                    | 0.797                         | 889                    | 11 248                                  | 1.395            | (18, 2)  | 1.515         | 5.21              | 1                | 1868                   | 5353                                    | 0.664                         | 958                    | 10 433                                  | 1.294            |
| (11, 0)  | 0.873         | 0.00              | 2                | 1037                   | 9644                                    | 1.196                         | 745                    | 13 431                                  | 1.665            | (18, 4)  | 1.611         | 9.83              | 2                | 1838                   | 5439                                    | 0.674                         | 1118                   | 8947                                    | 1.109            |
| (11, 1)  | 0.916         | 4.31              | 1                | 1265                   | 7906                                    | 0.980                         | 610                    | 16 388                                  | 2.032            | (18, 5)  | 1.663         | 11.93             | 1                | 2003                   | 4993                                    | 0.619                         | 1052                   | 9508                                    | 1.179            |
| (11, 3)  | 1.014         | 11.74             | 2                | 1197                   | 8353                                    | 1.036                         | 793                    | 12 617                                  | 1.564            | (18, 7)  | 1.773         | 15.75             | 2                | 2020                   | 4952                                    | 0.614                         | 1197                   | 8351                                    | 1.035            |
| (11, 4)  | 1.068         | 14.92             | 1                | 1371                   | 7295                                    | 0.904                         | 712                    | 14 036                                  | 1.740            | (18, 8)  | 1.831         | 17.48             | 1                | 2156                   | 4638                                    | 0.575                         | 1159                   | 8629                                    | 1.070            |
| (11, 6)  | 1.186         | 20.36             | 2                | 1397                   | 7157                                    | 0.887                         | 858                    | 11 661                                  | 1.446            | (18, 10) | 1.951         | 20.63             | 2                | 2218                   | 4509                                    | 0.559                         | 1288                   | 7765                                    | 0.963            |
| (11, 7)  | 1.248         | 22.69             | 1                | 1516                   | 6597                                    | 0.818                         | 836                    | 11 968                                  | 1.484            | (19, 0)  | 1.508         | 0.00              | 1                | 1867                   | 5357                                    | 0.664                         | 953                    | 10 492                                  | 1.301            |
| (11, 9)  | 1.377         | 26.70             | 2                | 1617                   | 6183                                    | 0.767                         | 947                    | 10 564                                  | 3.10             | (19, 2)  | 1.594         | 4.95              | 2                | 1816                   | 5507                                    | 0.683                         | 1114                   | 8979                                    | 1.113            |
| (11, 10) | 1.444         | 28.43             | 1                | 1702                   | 5877                                    | 0.729                         | 969                    | 10 320                                  | 1.280            | (19, 3)  | 1.641         | 7.22              | 1                | 1994                   | 5015                                    | 0.622                         | 1033                   | 9683                                    | 1.201            |
| (12, 1)  | 0.995         | 3.96              | 2                | 1170                   | 8549                                    | 1.060                         | 799                    | 12 516                                  | 1.552            | (19, 5)  | 1.741         | 11.39             | 2                | 1979                   | 5052                                    | 0.626                         | 1187                   | 8422                                    | 1.044            |
| (12, 2)  | 1.041         | 7.59              | 1                | 1378                   | 7255                                    | 0.900                         | 686                    | 14 575                                  | 1.807            | (19, 6)  | 1.795         | 13.29             | 1                | 2135                   | 4684                                    | 0.581                         | 1130                   | 8853                                    | 1.098            |
| (12, 4)  | 1.145         | 13.90             | 2                | 1342                   | 7452                                    | 0.924                         | 855                    | 11 693                                  | 1.450            | (19, 8)  | 1.907         | 16.76             | 2                | 2164                   | 4621                                    | 0.573                         | 1271                   | 7869                                    | 0.976            |
| (12, 5)  | 1.201         | 16.63             | 1                | 1499                   | 6670                                    | 0.827                         | 793                    | 12 605                                  | 1.563            | (19, 9)  | 1.966         | 18.35             | 1                | 2295                   | 4358                                    | 0.540                         | 1238                   | 8076                                    | 1.001            |
| (12, 7)  | 1.321         | 21.36             | 2                | 1545                   | 6473                                    | 0.803                         | 930                    | 10 751                                  | 1.333            | (20, 0)  | 1.588         | 0.00              | 2                | 1808                   | 5531                                    | 0.686                         | 1113                   | 8989                                    | 1.114            |
| (12, 8)  | 1.384         | 23.41             | 1                | 1657                   | 6036                                    | 0.748                         | 917                    | 10 910                                  | 1.353            | (20, 1)  | 1.629         | 2.42              | 1                | 1990                   | 5024                                    | 0.623                         | 1023                   | 9775                                    | 1.212            |
| (12, 10) | 1.515         | 27.00             | 2                | 1765                   | 5666                                    | 0.703                         | 1024                   | 9763                                    | 1.210            | (20, 3)  | 1.719         | 6.89              | 2                | 1952                   | 5124                                    | 0.635                         | 1181                   | 8466                                    | 1.050            |
| (12, 11) | 1.582         | 28.56             | 1                | 1848                   | 5412                                    | 0.671                         | 1049                   | 9535                                    | 1.182            | (20, 4)  | 1.768         | 8.95              | 1                | 2122                   | 4712                                    | 0.584                         | 1108                   | 9025                                    | 1.119            |
| (13, 0)  | 1.032         | 0.00              | 1                | 1384                   | 7228                                    | 0.896                         | 677                    | 14 770                                  | 1.831            | (20, 6)  | 1.872         | 12.73             | 2                | 2121                   | 4715                                    | 0.585                         | 1258                   | 7947                                    | 0.985            |
| (13, 2)  | 1.120         | 7.05              | 2                | 1307                   | 7651                                    | 0.949                         | 858                    | 11 661                                  | 1.446            | (20, 7)  | 1.927         | 14.46             | 1                | 2269                   | 4406                                    | 0.546                         | 1208                   | 8280                                    | 1.027            |
| (13, 3)  | 1.170         | 10.16             | 1                | 1498                   | 6676                                    | 0.828                         | 764                    | 13 095                                  | 1.624            | (21, 1)  | 1.708         | 2.31              | 2                | 1938                   | 5161                                    | 0.640                         | 1178                   | 8487                                    | 1.052            |
| (13, 5)  | 1.278         | 15.61             | 2                | 1487                   | 6723                                    | 0.834                         | 922                    | 10 843                                  | 1.344            | (21, 2)  | 1.752         | 4.50              | 1                | 2116                   | 4727                                    | 0.586                         | 1095                   | 9134                                    | 1.132            |
| (13, 6)  | 1.336         | 17.99             | 1                | 1632                   | 6127                                    | 0.760                         | 874                    | 11 441                                  | 1.419            | (21, 4)  | 1.847         | 8.57              | 2                | 2090                   | 4786                                    | 0.593                         | 1250                   | 8000                                    | 0.992            |
| (13, 8)  | 1.457         | 22.17             | 2                | 1692                   | 5909                                    | 0.733                         | 1004                   | 9956                                    | 1.234            | (21, 5)  | 1.897         | 10.44             | 1                | 2253                   | 4439                                    | 0.550                         | 1184                   | 8445                                    | 1.047            |
| (13, 9)  | 1.521         | 24.01             | 1                | 1799                   | 5558                                    | 0.689                         | 997                    | 10 027                                  | 1.243            | (22, 0)  | 1.747         | 0.00              | 1                | 2114                   | 4731                                    | 0.587                         | 1090                   | 9171                                    | 1.137            |
| (13, 11) | 1.652         | 27.25             | 2                | 1912                   | 5230                                    | 0.648                         | 1102                   | 9071                                    | 1.125            | (22, 2)  | 1.831         | 4.31              | 2                | 2070                   | 4830                                    | 0.599                         | 1245                   | 8030</td                                |                  |

closely. As before, different parameters apply according to whether  $\text{mod}(n - m, 3)$  equals 1 or 2 (mod 1 or mod 2 species, respectively). The refined empirical functions for first and second van Hove transition frequencies are (for  $d_t$  in nm)

$$\bar{\nu}_{11}(\text{mod } 1) = \frac{1 \times 10^7 \text{ cm}^{-1}}{157.5 + 1066.9d_t} - 771 \text{ cm}^{-1} \frac{[\cos(3\alpha)]^{1.374}}{d_t^{2.272}} \quad (1a)$$

$$\bar{\nu}_{11}(\text{mod } 2) = \frac{1 \times 10^7 \text{ cm}^{-1}}{157.5 + 1066.9d_t} + 347 \text{ cm}^{-1} \frac{[\cos(3\alpha)]^{0.886}}{d_t^{2.129}} \quad (1b)$$

$$\bar{\nu}_{22}(\text{mod } 1) = \frac{1 \times 10^7 \text{ cm}^{-1}}{145.6 + 575.7d_t} + 1326 \text{ cm}^{-1} \frac{[\cos(3\alpha)]^{0.828}}{d_t^{1.809}} \quad (2a)$$

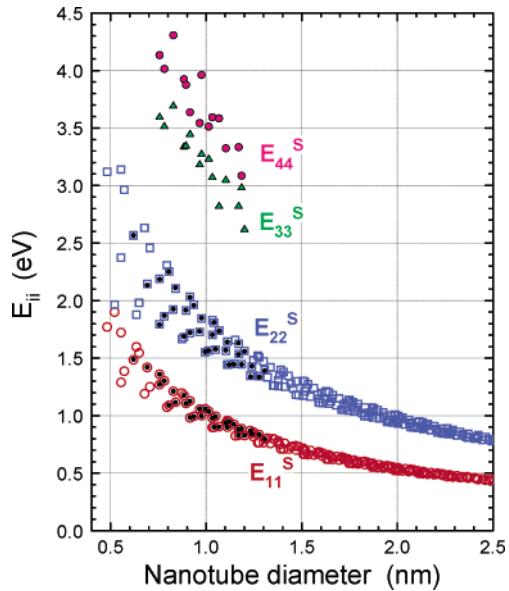
$$\bar{\nu}_{22}(\text{mod } 2) = \frac{1 \times 10^7 \text{ cm}^{-1}}{145.6 + 575.7d_t} - 1421 \text{ cm}^{-1} \frac{[\cos(3\alpha)]^{1.110}}{d_t^{2.497}} \quad (2b)$$

These expressions mimic our experimental findings with average errors of  $10 \text{ cm}^{-1}$  (1.3 meV) for  $\bar{\nu}_{11}$  and  $40 \text{ cm}^{-1}$  (5 meV) for  $\bar{\nu}_{22}$ . Compared to the previous fit with integer exponents, these expressions eliminate systematic fitting discrepancies and reduce average errors by approximately 70 and 40%. It seems likely that this accuracy will be retained on extrapolation to larger-diameter tubes, for which trigonal warping and curvature exert smaller spectral effects. Although extrapolation to smaller tube diameters is less secure, we expect the expressions to be useful down to ca. 0.5 nm.

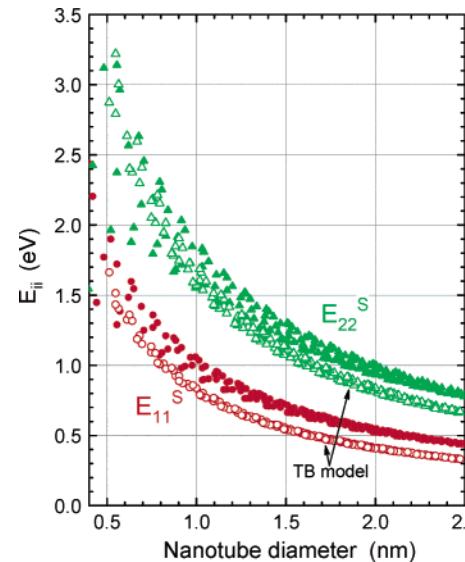
Table 1 lists all 127 semiconducting nanotube structures having diameters between 0.48 and 2.00 nm (assuming a C–C distance of 0.144 nm) in order of increasing  $n$  and  $m$  values. Following the columns that show the diameter, chiral angle, and  $\text{mod}(n - m, 3)$  values, we give the wavelengths, frequencies, and photon energies for first van Hove emission and second van Hove absorption as predicted by the equations above. Note that first van Hove absorption energies will be ca. 4 meV higher than the listed emission values.

In Figure 1, we plot the predicted and measured  $E_{11}$  and  $E_{22}$  values for semiconducting SWNT with diameters between 0.48 and 2.5 nm. A small set of measured values for  $E_{33}$  and  $E_{44}$  are also plotted. Empirical expressions for the energies of  $E_{33}$  and  $E_{44}$  transitions, which can also be important in resonance Raman studies of larger-diameter nanotubes, await further experimental measurements and analysis.

**Discussion.** Equations 1 and 2 are intended only as empirical fitting functions, and we do not suggest that the deduced parameters have any simple meaning within SWNT electronic structure models. Instead, the peculiar exponents deduced from fitting the data may reflect combinations of effects such as trigonal warping, curvature, and exciton binding.<sup>10–14</sup> Our fits and predictions apply to samples of individual SWNT in aqueous SDS suspension. Data measured by Lebedkin et al.<sup>9</sup> and in our laboratory suggest that small systematic spectral shifts of less than 2% can be



**Figure 1.** Optical transition energies vs diameter for semiconducting SWNT. Solid symbols are experimental data; open squares and circles are predictions of  $E_{11}$  and  $E_{22}$ , respectively, from the empirical fitting functions.



**Figure 2.** Comparison between model-based and empirical values of optical transition energies for semiconducting SWNT. Open symbols were computed using a simple tight binding (TB) model with  $\gamma_0 = 2.90 \text{ eV}$ , and solid symbols were obtained from the empirically based fitting functions.

expected for individual SWNT suspended in other aqueous surfactants. Note, however, that our spectral fits are not applicable to bundled suspended nanotubes, for which strong perturbations between tubes lead to significantly broadened and red-shifted absorption spectra and quenched band-gap luminescence.<sup>7</sup>

Although Figure 1 qualitatively resembles model-based results, there are significant quantitative differences. To illustrate these, Figure 2 shows plots of  $E_{11}$  and  $E_{22}$  computed from a simple tight-binding model (as commonly used for Kataura plots) overlaid with the empirically based values from Figure 1. Both for  $E_{11}$  and  $E_{22}$  transitions, the tight-

binding model badly underestimates the apparent scatter arising from chiral variations at a given diameter. Empirically, these chiral variations in transition energy are at least twice as large as predicted by the tight-binding model for tube diameters of 1.5 nm or less. It is also clear that the tight-binding model (with  $\gamma_0 = 2.90$  eV) underestimates  $E_{11}$  transition energies by up to 25% relative to the empirical values. Tight-binding  $E_{22}$  predictions are ca. 15% below the empirical values for 2.5-nm diameter tubes but as much as 25% *above* empirical values for some of the smallest-diameter species. Thus, adjusting  $\gamma_0$  cannot bring the simple model predictions into accord with the empirically based transition energies. In our view, these deficiencies in model-based Kataura plots appear large enough to cause serious interpretation problems in some optical nanotube investigations. We therefore recommend that researchers performing optical experiments with surfactant suspensions of SWNT instead rely on the findings reported here to guide their work.

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**Supporting Information Available:** We present as Supporting Information a version of Table 1 extended to include nanotube structures through (25, 24) and the  $E_{ii}$

values measured to date in room-temperature aqueous SDS suspensions. This material is available free of charge via the Internet at <http://pubs.acs.org>.

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